GRADE 6 COURSE OUTLINE

CONTENT AREA	CCSS STANDARDS	# OF DAYS
6.0 Introduction: Mathematical Investigations		5
6.1 Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.1, 2, & 3	10
6.2 Critical Area 1: 6.RP Critical Area 1: Connect ratio and rate to whole number multiplication and division and using the concepts of ratio and rate to solve problems		5
6.3 Apply and extend previous understandings of numbers to the system of rational numbers.	6.NS.5, 6, 7, & 8; 6.G.3	10
6.4 Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Compute fluently with multi-digit numbers	6.NS.1, 2, 3	10
6.5 Critical Areas 1 & 2: 6.RP, 6.NS Critical area 2: Complete understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers		13
6.6 Apply and extend previous understandings of arithmetic to algebraic expressions. Find common factors and multiples.	6.EE.1, 2, 3, & 4 6.NS.4	10
6.7 Reason about and solve one-variable equations and inequalities.	6.EE.5, 6, 7, & 8	10
6.8 Represent and analyze quantitative relationships between dependent and independent variables.	6.EE.9, 6.NS.8, 6.SP.4, 6.G.2	10
6.9 Critical Area 1,2, & 3: 6.RP, 6.NS, 6.EE Critical Area 3: Writing, interpreting and using expressions and equations		18
6.10 Develop understanding of statistical variability. Summarize and describe distributions.	6.SP.1, 2, 3, 4, & 5	10
6.11 Solve real-world and mathematical problems involving area, surface area, and volume.	6.G.1, 2, 3, & 4	9
6.12 Critical Area 1, 2, 3, & 4: 6.RP, 6.NS,: 6.EE, 6.SP Critical Area 4: Developing understanding of statistical thinking		24
	Total Days:	144

MIDDLE GRADES OVERVIEW

Green indicates a Smarter Balance Assessment Consortium *major* cluster, and yellow indicates an *additional or supporting* cluster.

GRADE 6	GRADE 7	GRADE 8
6.RP Understand ratio concepts and use ratio reasoning to solve problems.	7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.	8.EE Understand the connections between proportional relationships, lines, and linear equations.
6.EE Represent and analyze quantitative relationships between dependent and independent variables.		8.F Define, evaluate, and compare functions. 8.F Use functions to model relationships between quantities.
6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.	7.EE Use properties of operations to generate equivalent expressions.	8.EE Work with radicals and integer exponents.
6.EE Reason about and solve one-variable equations and inequalities.	7.EE Solve real-life and mathematical problems using numerical and algebraic equations.	8.EE Analyze and solve linear equations and pairs of simultaneous linear equations.
6. NS Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	7. NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	8.NS Know that there are numbers that are not rational, and approximate them by rational numbers.
6.NS Compute fluently with multi-digit numbers and find common factors and multiples.		
6.G Solve real-world and mathematical problems involving area, surface area, and volume.	7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G Draw, construct, and describe geometrical figures and describe the relationships between them.	8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
		8.G Understand congruence and similarity using physical models, transparencies, or geometry software. 8.G Understand and apply the Pythagorean theorem.
6.SP Develop understanding of statistical variability. 6.SP Summarize and describe distributions.	7.SP Use random sampling to draw inferences about a population. 7.SP Draw informal comparative inferences about two populations. 7.SP Investigate chance processes and develop, use, and evaluate probability models.	8.SP Investigate patterns of association in bivariate data.
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

CCSS TOOLS: PROGRESSIONS

Source: http://ime.math.arizona.edu/progressions/

GRADE 6	GRADE 7	GRADE 8
6.RP Understand ratio concepts and use ratio reasoning to solve problems.	7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.	8.EE Understand the connections between proportional relationships, lines, and linear equations.
6.EE Represent and analyze quantitative relationships between dependent and independent variables.		8.F Define, evaluate, and compare functions. 8.F Use functions to model relationships between quantities.
6.EE Apply and extend previous understandings of arithmetic to algebraic expressions. 6.EE Reason about and solve one-variable equations and inequalities.	7.EE Use properties of operations to generate equivalent expressions. 7.EE Solve real-life and mathematical problems using numerical and algebraic equations.	8.EE Work with radicals and integer exponents. 8.EE Analyze and solve linear equations and pairs of simultaneous linear equations.
6.NS Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions. 6.NS Compute fluently with multi-digit numbers and find common factors and multiples.	7.NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	8. NS Know that there are numbers that are not rational, and approximate them by rational numbers.
6.G Solve real-world and mathematical problems involving area, surface area, and volume.	7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 8.G Draw, construct, and describe geometrical figures and describe the relationships between them.	8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. 8.G Understand congruence and similarity using physical models, transparencies, or geometry software. 8.G Understand and apply the Pythagorean theorem.
6.SP Develop understanding of statistical variability. 6.SP Summarize and describe distributions.	7.SP Use random sampling to draw inferences about a population. 7.SP Draw informal comparative inferences about two populations. 7.SP Investigate chance processes and develop, use, and evaluate probability models.	8.SP Investigate patterns of association in bivariate data.

INTRODUCTION TO THE COURSE: MATHEMATICAL RELATIONSHIPS
5 DAYS

This introductory unit is designed to provide students with an opportunity to engage in worthwhile mathematical investigations; it has a number of related purposes. The most important of these is to let students draw heavily on the eight Mathematical Practices and establish their importance to this entire course. Through working with the mathematical practices, students will come to understanding the importance of seeing, doing, re-constructing, and supposing, in learning mathematics, and hopefully also realize that mathematics is not just facts to be memorized.

Examples of the kinds of mathematical investigations that students will work on during this unit can be found at Henri Piciotto's Mathematics Education Page (http://www.mathedpage.org/).

One of these investigations is called "Area on Graph Paper"; it explores the least and greatest perimeters that can be made with polyominoes of a fixed area. This calls on students to make and test hypotheses, be systematic, and draw conclusions. It will encourage students to deploy MP.1 (*Make sense of problems and persevere in solving them*) and MP.7 (*Look for and make use of structure*). (http://www.MathEdPage.org/new-algebra/new-algebra.html#graph)

The McNuggets Problem, another type of mathematical investigation, which can also be found on Henri's website is particularly valuable in an introductory unit such as this. Students are invited to consider the number of chicken nuggets that might be bought if a fast food store sells them in quantities of 6, 9, and 20. Students are asked to determine how many nuggets can and cannot be bought. Students are also invited to determine what is the largest number of nuggets that cannot be bought, and then prove that every number greater than this largest number can be bought. (http://www.mathedpage.org/early-math/early.html)

Other interesting investigations can be found at Henri's Website. For example, in his paper *Operation Sense, Tool-Based Pedagogy Curricular Breath: A Proposal* he describes investigations such as *Perimeter Patterns* and *Angles* which can be enacted using Pattern Blocks (http://www.mathedpage.org/early-math/early.html#Fun-Math)

Another purpose of this introductory unit is to establish the norms for doing mathematics, and set the scene for the rest of the course. Teachers will be encouraged to enact these investigations in a way that enables students to take responsibility for their own learning and act as instructional resources for each other. The teacher will provide feedback that moves the learning forward and engineer effective discussion so as to facilitate learning. The intent of each mathematical investigation will be made explicit so as to provide a purpose for students.

The genius of this type of investigation is that, in a heterogeneous class of students, it is highly likely that every student will be able to do something, but also highly likely that no student will be able to do everything.

6.1 RATIO AND PROPORTIONAL RELATIONSHIPS
10 DAYS

This is one of those areas where the *CCSS* is calling for a major shift. Students' study of ratio in grade 6 underpins their work with proportional and all linear relationships in later grades. When we find the *ratio* of one number, *a*, to another number, *b*, by dividing *a* by *b,* we compare the quantities using division. The *CCSS* asks students do this in a variety of situations. Students learn that the same ratio can be represented in different ways (for example, .75, 75%, 3:4, 3 to 4, or $\frac{3}{4}$) and that a ratio can show the relationship between quantities measured with the same units or with different units. Further, for some situations, the ratio of one quantity to another may change depending on the circumstances (the ratio of the number of miles a car travels for each gallon of gas changes depending on where and how the car is driven), while for others (the relationship between any circle's circumference and diameter), the ratio remains constant. It is important to notice how the *CCSS* shifts emphasis away from setting up a proportion and solving it by cross-multiplication. A major emphasis of the *CCSS* is finding the unit rate.

Ratios and Proportional Relationships 6.RP
Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

3a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

3b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

3c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

3d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

[1]Expectations for unit rates in this grade are limited to non-complex fractions.

TEACHING AND ASSESSMENT RESOURCES

H. Hironaka & Y. Sugiyama, eds. *Mathematics 5B for Elementary School*. Tokyo, Japan: Tokyo Shoseki, 2006. (ISBN 4-487-46619-9; distributed by Global Education Resources, www.globaledresources.com)

Chapter 9: Per Unit Quantity, pp. 18–37.

H. Hironaka & Y. Sugiyama, eds. *Mathematics 6A for Elementary School*. Tokyo, Japan: Tokyo Shoseki, 2006. (ISBN 4-487-46620-2; distributed by Global Education Resources, www.globaledresources.com)

Chapter 5: Ratio and Its Value, pp. 48–59.

6.2 CRITICAL AREA
15 DAYS

By working on cross-cutting problems, involving multiplication, division, ratio, rate, fractions, and percent, students have the opportunity to integrate their new learning of Grade 6 with their early elementary learning of multiplication and division. This critical area of focus offers students a site for deep learning as they develop an understanding of comparison of numbers and quantities by division.

Critical Area 1
Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

Grade 6 students could even be called upon to analyze the so-called Buffett Rule. Why it is important to use ratio to compare the amount of tax paid by Mr. Buffett and his administrative assistants rather than using subtraction?

TEACHING AND ASSESSMENT RESOURCES FOR CRITICAL AREA 1

H. Hironaka & Y. Sugiyama, eds. *Mathematics 5B for Elementary School*. Tokyo, Japan: Tokyo Shoseki, 2006. (ISBN 4-487-46619-9; distributed by Global Education Resources, www.globaledresources.com)

Chapter 9: Per Unit Quantity, pp. 18–37

6.3 RATIONAL NUMBER SYSTEM
10 DAYS

In this section of the grade 6 course, students apply and extend previous understandings of numbers to the system of rational numbers. The priorities here are using the number line and the coordinate plane to represent and understand rational numbers. In their work with the coordinate plane, students can address the section of the *CCSS* that asks students to draw polygons in the coordinate plane given coordinates for vertices; to find the lengths of vertical and horizontal lines; and to apply these techniques in the context of solving real-world and mathematical problems. In this unit students also study the absolute value of a number as the number's distance from zero.

The Number System 6.NS

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

6a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.

6b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

6c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7. Understand ordering and absolute value of rational numbers.

7a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right*.

7b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C*.

7c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars*.

7d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars*.

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Geometry 6.G

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

(In this unit, students should be asked to order rational numbers expressed as fractions. Recently, we noticed that all but one student in a class of students ranked $\frac{8}{10}$ as smaller than $\frac{3}{4}$. Students justified this by arguing that while both were really close to 1, the fraction $\frac{3}{4}$ was closer to 1 because 3 is "only one away from 4," but 8 is "two away from 10." Arguments such as these show that some students do not have a robust sense of fraction. One student in the class ranked $\frac{3}{4}$ as smaller than $\frac{8}{10}$, but he had a difficult time justifying his correct conclusion. He did argue that $\frac{8}{10}$ could be simplified to give $\frac{4}{5}$, but did not say how this helped to furnish a justification for his correct conclusion.)

TEACHING AND ASSESSMENT RESOURCES

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/

6.4 DIVISION OF FRACTIONS BY FRACTIONS
10 DAYS

Division of fractions will be considerably easier if students are given the opportunity to develop a robust notion of fraction. In our work with students, we found that they can do the most complicated computations with fractions by hand, but have little idea of the meaning of individual fractions. Recently, students were asked to rank seven fractions, three of which were $\frac{1}{25}, \frac{1}{4}$, and $\frac{1}{8}$. We watched in awe as students painstakingly converted each to a decimal, and wondered why students had not just thought about the meaning of these unit fractions.

The Number System 6.NS
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.
2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.

Later, the same students were asked to match the same seven fractions with seven area models. The area model that represented $\frac{1}{4}$ had two out of eight partitions shaded. Many students concluded that there was no match for $\frac{1}{4}$. Others concluded that since $\frac{2}{8}$ could be simplified to give $\frac{1}{4}$, the area model showing two out of 8 partitioned shaded was a correct match. Even so, students were unable to show us how $\frac{1}{4}$ "showed up in the area model." For example, when they matched $\frac{1}{25}$ with its area model they could say easily how $\frac{1}{25}$ "showed up in its area model" by explaining that there were 25 squares and just one shaded. However, no student could use the structure of the area model to justify matching $\frac{1}{4}$ with $\frac{2}{8}$. When we gave students the task of "finding the one and finding the four," all the students were preoccupied with computations. Finally, we heard one student say, Hey, we don't have to do the math, we just have to think about it.

TEACHING AND ASSESSMENT RESOURCES

H. Hironaka & Y. Sugiyama, eds. *Mathematics 6A for Elementary School*. Tokyo, Japan: Tokyo Shoseki, 2006. (ISBN 4-487-46620-2; distributed by Global Education Resources, www.globaledresources.com)

Chapter 1: Multiplication and Division of Fractions, pp. 4–6
Chapter 2: Multiplication of Fractions, pp. 7–16
Chapter 3: Division of Fractions, pp. 17–29

6.5 CRITICAL AREAS 1 AND 2
13 DAYS

In this part of the course, students get to pull together the mathematics of the first three units. The focus here is on tackling performance tasks that cut across the mathematics that they have studied so far.

It is fair that students be asked to address problems that draw on K-5 mathematics—without review.

Critical Area 2
Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Critical Area 1
Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

TEACHING AND ASSESSMENT RESOURCES

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/

6.6 EXPRESSIONS
10 DAYS

The following source provides a grade 6 through grade 8 progression for Expressions and Equations: http://ime.math.arizona.edu/progressions/ This document is useful in that it integrates the mathematical practices and the content standards. It also draws attention to common errors.

Opportunities to generalize are essential for students in grade 6. When students write and evaluate numerical expressions involving whole-number exponents, they will learn that $3 \cdot 3$ can be written as 3^2. At this time also students need to consider how $n \cdot n$ could be written. Recently we observed as a group of grade 8 students struggled to compute $b \cdot b$. Students debated if it should be 1*b*, 2*b*, or *bb*. Finally, it dawned on one student that $b \cdot b$ should be written as b^2, and then that seemed obvious to all of the students. Frequent opportunities to generalize arithmetic will benefit all students when they embark on polynomial arithmetic.

Expressions and Equations 6.EE
Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
2a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
2b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
2c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

The Number System 6.NS
Compute fluently with multi-digit numbers and find common factors and multiples.

4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

TEACHING AND ASSESSMENT RESOURCES

Kunihiko Kodaira et al. Japanese Grade 7 Mathematics. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

Chapter 1: Factors and Multiples, pp. 1–18

Chapter 3: Letters and Expressions, pp. 55–65

6.7 EQUATIONS AND INEQUALITIES
10 DAYS

The following source provides a grade 6 through grade 8 progression for expressions and equations: http://ime.math.arizona.edu/progressions/ This document is useful in that it integrates the mathematical practices and the content standards. It also draws attention to common errors.

In working with equations, students will learn that an equation consists of two expressions linked by an equals sign and will be able to identify examples and non-examples of equations. Students must understand that solving an equation in one unknown means finding a value of the letter that makes the left side equal to the right side. In working with equations, we recommend not introducing the term variable. Instead, we recommend just reminding students that in mathematics letters are used to stand for numbers.

Expressions and Equations 6.EE
Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

TEACHING AND ASSESSMENT RESOURCES

Kunihiko Kodaira et al. Japanese Grade 7 Mathematics. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

Chapter 4: Equations, pp. 76–83

6.8 QUANTITATIVE RELATIONSHIPS
10 DAYS

The following source provides a grade 6 through grade 8 progression for expressions and equations: http://ime.math.arizona.edu/progressions/ This document is useful in that it integrates the mathematical practices and the content standards. It also draws attention to common errors.

This part of the course should begin with an intuitive analysis of how one varying quantity changes in relation to another varying quantity. This intuitive visualization work is essential because all too often students engage with the mechanics of computing or graphing. Here are some contexts that students might think about, imagine, or sketch on a graph:

- How the height of water changes in a cylinder as more water is poured into the cylinder;
- Given rectangles of fixed area, how the length changes as the width is increased;
- As the weight on the end of a spring increases, how the length of the spring changes;
- How the length of a candle changes as burning time increases.

Expressions and Equations 6.EE
Represent and analyze quantitative relationships between dependent and independent variables.
9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time*.

These contexts will allow students to think about quantitative relationships across many different contexts, and enable them to connect various aspects of mathematics that they might have kept separate.

The Number System 6.NS
Apply and extend previous understandings of numbers to the system of rational numbers.
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Students can be asked to graph the volume of liquid in a cylinder in terms of the height of the liquid. This will cause students to integrate the idea, concepts, and skills called for in the *CCSS* (6.EE, 6.NS.8, 6.G.2).

Statistics and Probability 6.SP
Summarize and describe distributions.
4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

In such contexts students will be also called upon to address the Mathematical Practices, in particular MP.2 (Reason abstractly and quantitatively), MP.5 (Use appropriate tools strategically), and MP.6 (Attend to precision).

Geometry 6.G
Solve real-world and mathematical problems involving area, surface area, and volume.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

TEACHING AND ASSESSMENT RESOURCES

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Chapter 9: Per Unit Quantity, pp. 18–37

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Chapter 7: Various Ways of Changing, pp. 70–71

H. Hironaka & Y. Sugiyama, eds. *Mathematics for Elementary School 6B.* (ISBN 4-487-46621-0; distributed by Global Education Resources, www.globaledresources.com)

Chapter 14: Various Graphs, pp. 61–65

6.9 CRITICAL AREAS 1, 2, AND 3
18 DAYS

This part of the course is designed to create for students the opportunity to deepen their learning. It is not designed as an opportunity to introduce more content or do more practice exercises. This is designed as site for deepening and consolidating learning.

Critical Area 3
Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as 3x = y) to describe relationships between quantities.

Critical Area 2
Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Critical Area 1
Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

TEACHING AND ASSESSMENT RESOURCES

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/

6.10 STATISTICS
10 DAYS

The key to beginning well in statistics is formulating appropriate questions. Peer editing of questions can be a valuable experience for students in refining their questions. In order for students to get meaning out of their data, they must connect the question with the data, the data with a graph, and the graph and data with a summary, and then connect that summary back to the question. This unit begins to develop in students a **number sense** for data and statistics. This unit builds the groundwork for success in statistics later in the curriculum.

Statistics and Probability 6.SP
Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:

5a. Reporting the number of observations.
5b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
5c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
5d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

TEACHING AND ASSESSMENT RESOURCES

H. Hironaka & Y. Sugiyama, eds. *Mathematics for Elementary School 6B.* (ISBN 4-487-46621-0; distributed by Global Education Resources, www.globaledresources.com)

Chapter 12: How to Analyze Data, pp. 44–52

Novice Tasks	Apprentice Tasks	Expert Tasks
	A25: Suzi's Company	

6.11 GEOMETRY
9 DAYS

In this section it is critically important that students work is hands-on and visceral. Students need the opportunity to find the volume of solids, create solids, and then make nets to support the calculation of surface area. Students can also use the nets to derive formulas for the surface area of three-dimensional figures.

Geometry 6.G
Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

TEACHING AND ASSESSMENT RESOURCES

Henri Piccotto. *Geometry Labs.* 1999. (*Geometry Labs* can be downloaded from www.mathedpage.org for non-commercial use. Henri Piccotto grants the teacher who downloads Geometry labs the right to reproduce materials for any non-commercial use.)

8. Perimeter and Area
   - Lab 8.1: Polyomino Perimeter and Area, pp. 106–108
   - Lab 8.2: Minimizing Perimeter, pp. 109–110
   - Lab 8.3: A formula for Polyomino Perimeter, pp. 111–112
   - Lab 8.4: Geoboard Area, pp. 113–114
   - Lab 8.5: Geoboard Squares, pp. 115
   - Lab 8.6: Pick's Formula, pp. 116–17

H. Hironaka & Y. Sugiyama, eds. *Mathematics for Elementary School 6B.* (ISBN 4-487-46621-0; distributed by Global Education Resources, www.globaledresources.com)

Chapter 9: Solid Figures, pp. 4–18
Chapter 10: Surface Area and Volume of Solid Figures, pp. 19–30

Novice Tasks	Apprentice Tasks	Expert Tasks
		E01: Candle Box
		E08: Smoothie Box

6.12 CRITICAL AREAS 1, 2, 3, AND 4
24 DAYS

It is in this section that students pull the mathematics of their grade 6 course together. Research has shown that students learn more by working on a few challenging problems than by completing many simple exercises.

Critical Area 4

Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Here, the students continue to make connections between the summary of a distribution and the graph of a distribution. Too often, students fail to connect their graphs or summaries back to the original question or scenario. Statistics is not just a study of numbers, but it is a study of numbers *in context*.

Critical Area 3

Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as 3x = y) to describe relationships between quantities.

Students grapple with challenging problems that cut across all of grade 6 and below. Many challenging problems can be set so that students have plenty of practice thinking and figuring out how to get started on multi-step problems.

Critical Area 2

Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Math problems that draw upon the concepts of K-6 can be posed here. Students can also be charged with creating their own math problems or creating their own study guides. Engaging students in a productive struggle with mathematics is known to be an effective review strategy.

Critical Area 1

Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

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